CAPT Anthony (Tony) Cowden, USN
2 March 2021

Subj: A STUDY GROUP: THE SALVO EQUATIONS (U)

(U) EXECUTIVE SUMMARY

1. (U) A study group was formed to introduce interested Staff officers to the Salvo Equations, use the Salvo Equations to analyze a relevant operational scenario, and develop applicable lessons learned:
    a. Numbers matter.
    b. The marginal value of additional combat power can be significant.
    c. Reload requirements/time can be critical.
    d. The Fleet's operational reserve is in its "base".
    e. Effective scouting is critical.
    f. Limitations to the use of the Salvo Equations.

(U) BACKGROUND

2. (U) A study group was formed for the purpose of introducing interested Staff officers to the Salvo Equations, first developed by CAPT Wayne Hughes, author of *Fleet Tactics*.[1] The Study Group consisted of Captain Anthony Cowden, LT Audrey Michelli, and LT Nicholas Junker. The reference for the analysis was the soon to be published *Fighting the Fleet: Operational Art and Modern Fleet Combat*.[2]

(U) DISCUSSION

3. (U) The first task undertaken by the group was to become familiar with the Salvo Equations and their usage as described in Chapter 2 of *Fighting the Fleet.* The Salvo Equations are presented here for reference:

$$\Delta A = \frac{\beta B - a_3 A}{a_1} \quad , \quad \Delta B = \frac{\alpha A - b_3 B}{b_1}$$

where:

A = number of units in force A.
B = number of units in force B.

[1] Hughes, Wayne P. and Robert Girrier. *Fleet Tactics and Naval Operations, Third Edition.* Annapolis, MD: Naval Institute Press, 2018.
[2] Cares, Jeffrey R. and Anthony Cowden. *Fighting the Fleet: Operational Art and Modern Fleet Combat.* Annapolis, MD: Naval Institute Press, Fall 2021.

β = number of well-aimed missiles fired by each B unit.
α = number of well-aimed missiles fired by each A unit.
a_1 = number of hits by B's missiles needed to put one A out of action.
b_1 = number of hits by A's missiles needed to put one B out of action.
a_3 = number of well-aimed missiles destroyed by each A.
b_3 = number of well-aimed missiles destroyed by each B.
ΔA = number of units in force A out of action from B's salvo.
ΔB = number of units in force B out of action from A's salvo.

4. (U) The next task was to implement the Salvo Equations in a spreadsheet and use them to represent a hypothetical engagement between two opposing surface combatant forces.[3] Parameters and assumptions associated with this scenario include the following:
    a. All ship and weapon system characteristics were derived from UNCLASSIFIED sources (i.e., Wikipedia).
    b. Ship classes and weapon systems were based on the ARLEIGH BURKE-class DDG and GRIGOROVICH-class FFG.
    c. Initial force size was 2 x Blue Force DDGs versus 2 x Red Force FFGs.
    d. Ship weapon loadouts were as follows:
        1. For the DDGs (Blue Force B):
            a. Offensive weapons consisted of:
                i. 8 x Harpoon Anti-Ship Cruise Missiles (ASCMs)
                ii. 18 x SM-6 missiles used in the anti-surface mode
            b. Defensive weapons consisted of:
                i. 1 x Phalanx Close-In Weapon Systems (CIWS)
                ii. 18 x SM-2 Surface-to-Air Missiles (SAMs)
                iii. 16 x Evolved Sea Sparrow Missiles (ESSMs).
        2. For the FFGs (Red Force A):
            a. Offensive weapons consisted of:
                i. 4 x Kalibr M
                ii. 4 x P-800 Oniks ASCMs
            b. Defensive weapons consisted of:
                i. 2 x AK-630M-2 CIWS
                ii. 24 x 9M317M SAMs
    e. Assumptions:
        1. Unless otherwise indicated, engagements occurred near simultaneously with similar scouting capabilities.[4]
        2. There were no other vessels in the target areas, such that ASCMs did not accidentally strike other vessels instead of the intended target.

---

[3] While not specifically used by the Study Group in this effort, Appendix B of *Fighting the Fleet* contains some thoughts on how to use the Salvo Equations in force planning.

[4] "Simultaneity" in a salvo exchange exists as long as each force gets off its offensive strike before receiving damage from the other force. Recall that the Japanese aircraft carriers that launched the strike that crippled USS YORKTOWN at the Battle of Midway had been sunk themselves before their pilots returned from their strike.

3. It takes two SAMs to defeat 1 x ASCM.
4. In terms of **a3** and **b3** (the number of well-aimed missiles destroyed by Force A and B ships, respectively):
    a. A ship's defensive SAM system was able to defeat two (2) incoming ASCMs.
    b. A ship's passive and non-kinetic defensive systems (decoys, jamming, etc.) could defeat two (2) ASCMs.
    c. All CIWS are able to defeat 2 x ASCMs before their magazine was expended. Note that each Blue Force DDG was equipped with 1 x CIWS and each Red Force FFG was equipped with 2 x CIWS. This results in **a3 = 8** being greater than **b3 = 6.**
5. It takes two (2) ASCMs to knock a Red Force FFG out of action **(a_1)**, and three (3) ASCMs to knock a Blue Force DDG out of action **(b_1)**, due to the relatively larger size of the DDG.

5. (U) Based on the assumptions outlined above, the Salvo Equations for an engagement between the 2 x Blue Force DDGs and the 2 x Red Force FFGs are contained in **Figure 1**.

$$\Delta A = \frac{(10)(2) - (8)(2)}{2} = \frac{(20 - 16)}{2} = 2$$

$$\Delta B = \frac{(8)(2) - (6)(2)}{3} = \frac{(16 - 12)}{3} = 1.33$$

Figure 1. Engagement Salvo Equations

6. (U) Interpretation of the results of this engagement are as follows:
    a. Firing all of its offensive missiles (**α = 8)**, Red Force A could expect to achieve extensive damage to both Blue Force B ships**.** [5]
    b. Blue Force B, on the other hand, could expect to knock out both Red Force FFGs (**ΔA** = 2) by each ship firing only 10 of its 26 offensive missiles (**β = 10)**. Thought should be given at the tactical level, however, to the composition of the salvo in terms of missile type. For example, the Harpoon has a much larger warhead and employs an attack profile designed to sink a ship, while the SM-6 is a much faster missile with a much smaller warhead.
    c. A prudent force planner would want to know the marginal effect of shooting slightly fewer or slightly more offensive missiles – to see if they might economize on weapons or, alternatively, how many more missiles they might need to shoot for a more comfortable margin of victory. **Figure 3** presents the results of Blue Force B firing two fewer offensive missile per ship,

[5] Damage is spread across the entire force, and not necessarily evenly. The term **ΔB** indicates the total damage suffered by the entire force.

d. **Figure 4** presents the results of firing one fewer offensive missile per ship, **Figure 4** presents the results of firing one more offensive missile per ship, and **Figure 5** presents the results of firing two more offensive missile per ship.

$$\Delta A = \frac{(8)(2) - (8)(2)}{2} = \frac{(16 - 16)}{2} = 0$$

Figure 2. Blue Fires Two Fewer Missiles per Ship

$$\Delta A = \frac{(9)(2) - (8)(2)}{2} = \frac{(18 - 16)}{2} = 1$$

Figure 3. Blue Fires One Fewer Missile per Ship

$$\Delta A = \frac{(11)(2) - (8)(2)}{2} = \frac{(22 - 16)}{2} = 3$$

Figure 4. Blue Fires One Additional Missile per Ship

$$\Delta A = \frac{(12)(2) - (8)(2)}{2} = \frac{(24 - 16)}{2} = 4$$

Figure 5. Blue Fires Two Additional Missiles per Ship

(U) The result of these iterations is depicted in **Figure 6**. This shows compelling evidence of a knife's edge distribution of outcomes: the original salvo size of 10 missiles per ship is "just right", but one less is far too little (Force A actually comes out ahead in the exchange), and one more missile per ship provides a substantial margin of victory.

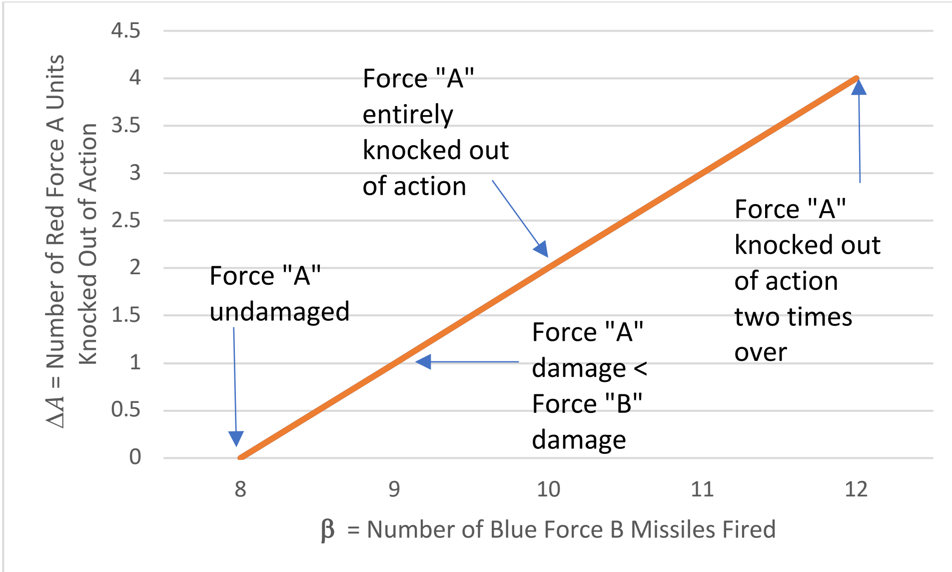

Figure 6. Force "A" Damage as a Function of the Number of Force "B" Missiles

7. (U) If Red Force A was conducting this analysis, they might decide that they sent too small a force against the two Blue Force DDGs, so the next analysis conducted was to add a third Red Force FFG to Force A. **Figure 7** represents the results of adding a third Red Force FFG:

$$\Delta A = \frac{(15)(2) - (8)(3)}{2} = \frac{30 - 24}{2} = 3$$

$$\Delta B = \frac{(8)(3) - (6)(2)}{3} = \frac{(24 - 12)}{3} = 4$$

Figure 7. 2 x Blue Force DDGs vs. A Third Red Force FFG

(U) The results of adding a third Red Force FFG show that the Blue Force DDGs must now fire 15 offensive missiles *each* in order to knock out all three (3) Red Force FFGs. However, the combined offensive power of the 3 x Red Force FFGs results in the 2 x Blue Force DDGs being knocked out of action two times over. This shows definitively that numbers count in salvo warfare.

8. (U) Assuming that the first engagement has occurred and that later on, before the Blue Force DDGs have been able to be repaired and rearmed, a single Red Force FFG attacks the two Blue Force DDGs, then the engagement of the two damaged Blue Force DDGs with an undamaged Red Force FFG would result in the Salvo Equations depicted in **Figure 8** . In this case, Force B consists of the two damaged (2) Blue Force DDGs, with their individual combat power reduced by 2/3's. This is accomplished by reducing the number of hits it would take to knock each ship out of action **(b1)** from three (3) to one (1), the number of inbound missiles it can counter **(b3)** from six (6) to two (2), and the remaining offensive

missiles from 16 to five (5). In this engagement, the Red Force FFG would be sunk, but so would be the two (2) Blue Force DDGs.

$$\Delta A = \frac{(5)(2) - (8)(1)}{2} = \frac{(10 - 8)}{2} = 1$$

$$\Delta B = \frac{(8)(1) - (2)(2)}{1} = \frac{(8 - 4)}{1} = 4$$

Figure 8. 2 x Damaged Blue Force DDGs vs. 1 x Undamaged Red Force FFG

(U) LESSONS LEARNED

- **(U) Numbers matter.** With only eight (8) offensive missiles per ship, Red Force A was not able to fully overcome Blue Force B's defensive capability. However, if each Red Force A ship had only one (1) more missile, it could expect to knock both Blue Force B ships out of action. And as we can see in **Figure 7**, the addition of one more Red Force FFG allows Red Force A to handily destroy Blue Force B and have a greater likelihood of part of the force surviving.[6] Doctrinally speaking, in order to prevail one must "pay the tax" of the opponents ability to defend against your offensive capability, and employ enough offensive power to overwhelm the enemy's defenses and achieve a robust outcome.
- **(U) The marginal value of additional combat power can be significant.** As noted above, the firing of 10 out of 26 offensive missiles from each Blue Force DDG should be enough to defeat the two Red Force FFGs, but firing just one more missile from each ship significantly added to the chances of victory.[7] This is, of course, because the earlier missiles forced the Red Force FFGs to expend their defensive weapons, leaving the field clear for the marginal addition of just a few missiles to get through. Additional defensive combat power should not be overlooked, either, but is harder to achieve: neither the Fleet commander nor the ship CO can add defensive weapon systems (only the design and acquisition process can do that) or increase a ship's staying power (i.e., the number of missiles required to knock a ship out of action).[8]
- **(U) Reload requirements/time can be critical.** In our analysis we assumed each CIWS could destroy two incoming missiles, but then its magazine had to be reloaded. If the magazine could be reloaded before a second salvo arrived, then the CIWS could destroy

[6] Combat entropy works both ways: a **ΔB** value of 2 suggests that both ships in the force will be knocked out of action, but just barely so – if things go just a little bit better for Force A, one or more of their ships, while being heavily damaged, might survive the fight.

[7] Note that the addition of the SM-6 to the US Navy's inventory is only a recent development. Until very recently the most long-range (over the horizon) anti-ship missiles a US Navy surface ship employed were eight (8) Harpoons. Without the SM-6 in these scenarios, the Blue Force ships would have decisively lost the initial engagement ($\boldsymbol{\Delta A} = 0$ *and* $\boldsymbol{\Delta B} = 1.33$).

[8] That being said, one of the advantages the US Navy had over the Imperial Japanese Navy in World War II was superior damage control, which often allowed damaged ships to continue fighting and heavily damaged ships to successfully return to repair facilities (see the lesson learned about the "base"). Effective damage control is an element within the Salvo Equations.

another two missiles. Therefore, it is better to shoot a single large salvo of offensive missiles than two smaller salvoes spaced in time greater than the time it takes to reload the CIWS. In the design and acquisition of ships and their combat systems, this also argues for CIWS with larger magazines, possibly RAM or SeaRAM, or CIWS systems that can be reloaded faster.

- **(U) The Fleet's operational reserve is in its "base".** In this usage, the term "base" means the whole of effort associated with replacing or repairing the damaged Fleet's reduced combat capability. Even if ships are undamaged, they must be refueled and rearmed; if damaged, their combat capability must be repaired or they run the risk of being involved in a future engagement, as we see in **Figure 8**, at less than full capability. Good offensive coordinators in football are not interested in *how* a play is conducted (tactical performance), only in the *outcome* of the play (operational result), so that they know what play (operational order) to send in next – doctrinally speaking, one must always be planning for the *next* engagement.
- **(U) Effective scouting is critical.** In our analysis we assumed that all ships could effectively target their offensive missiles out to their maximum effective range. This is no mean feat, especially if neutral shipping is present. *The side that gets off an unanswered salvo has the advantage* in that it may knock out or damage its opponent first (i.e., reduce its offensive and defensive combat capabilities) without receiving any damage or expending any defensive capabilities themselves. This is the powerful effect of Fire Effectively First. For example, if Red Force A had been able to get off an unanswered salvo against Blue Force B, it could expect to face a reduced Blue Force B consisting of two heavily damaged ships without suffering any damage itself. Providing a surface force with increased scouting resources to ensure they can "fire effectively first" is a critical function of Fleet planning.
- **(U) Limitations.** Care should be taken in not applying the Salvo Equations too "tactically": a necessary assumption is that individual ships are employing their weapon systems in accordance with their training and doctrine. The Salvo Equations are not intended to answer all the questions about a particular engagement; they are comparative in nature, not predictive, and are best employed at the coarse scale of operational planning. They do, however, have an important role to play in force planning to help answer some broad questions at the operational level of war.